A NEW SCHEMA FOR COMBINING NUMERICAL AND ANALYTICAL METHODS FOR RIGOROUS ANALYSIS OF WAVEGUIDE CIRCUITS

Ke-Li Wu *, Chen Wu ** and John Litva **

* COM DEV Ltd., 155 Shelton Drive, Cambridge, ON, Canada N1R 7H6

** Communications Research Lab., McMaster University, Ontario, Canada, L8S 4K1

Abstract

A new schema is proposed for rigorous analysis of waveguide circuits which are composed of a number of discontinuities. The schema combines numerical techniques with analytical techniques, such as the mode-matching method. The concept is the following; for a general complicated segment of waveguide circuit, a numerical technique is used to generate the multi-mode general scattering matrix, while the mode-matching technique is use to deal with all the regular discontinuities. The final result is obtained by cascading the general scattering matrices. To validate the schema, a numerical example is given. The result obtained with the new schema agrees very well with results obtained using full numerical techniques and experiment results.

1. Introduction

The analysis of rectangular waveguide discontinuities is always a hot topic in guided wave theory. This is not only because of its wide range of applications, such as beam forming networks in satellite communications systems and various waveguide components in radar systems, but also, because increasingly advanced systems need to be optimally designed, which requires the use of efficient analytical tools. There is no doubt that the most efficient tool for analysis is one based on closed form expressions, even though only a limited number of problems can be solved in this manner. Alternatively, numerical analysis can, in general, solve all types of waveguide discontinuities. A schema of combining numerical and analytical methods is proposed to enhance the applicability of analytical methods and to improve the efficiency of conventional numerical methods.

In the past few years, many numerical and analytical tools have been developed for field analysis of waveguide components. For example, the mode-matching method [1], the finite element method [2], the boundary element method [3], and recently, the coupled finite-boundary element method [4,5] are some of the more useful techniques that have been developed for analysis of practical components.. Among all the available methods, the mode-matching method (an analytical approach) is the most efficient and is highly recommended if the geometry of the discontinuity permits its use, that is, in those cases where the homogeneous waveguide junction is limited to regular shapes.

The question that arises naturally is whether one can develop a multi-mode scattering matrix for a highly composite and irregularly-shaped key-block using a general numerical method, while still making every effort to solve other regular segments using analytical approaches, e.g. the mode matching technique, and finally link all of the blocks by the well known generalized S-matrix technique? This paper will try to demonstrate that the answer is yes. The coupled finite-boundary element method (CFBM) [4] is used in this study. In principle, the same idea can be extended to other numerical techniques. The multi-mode scattering matrix rigorously takes into account mutual coupling between each of the key-building blocks, provided that a sufficient number of the modes is considered. Therefore, the overall analysis is still full wave in concept.

The theory is verified by showing good agreement between the result derived using the new schema with experimental results and those obtained using full numerical methods. It will be shown that the proposed new schema gives the same level of accuracy as that given by full-wave numerical analysis, but with a much reduced computational overhead.

2. Theoretical Formulation

In terms of their key building-block discontinuities, most waveguide circuits can be decomposed into two categories: the regular and the irregular. The regular discontinuities, such as step junctions, can be treated optimally by the mode matching technique. While, the irregular discontinuities, which are difficult to solve by the mode matching technique, can be solved using numerical methods. The bridge that links these two categories is the general scattering matrix. Since the interaction between higher order modes is accounted for in the general scattering matrix, mutual coupling between discontinuities is therefore dealt with in a rigorous manner.

Numerical Solutions:

For an arbitrarily shaped waveguide discontinuity, the coupled finite element and boundary element method (CFBM) is modified so as to generate a general scattering matrix. This hybrid method merges the advantages of the finite element and boundary element methods; therefore, it can handle fairly complicated waveguide problems. Going a step further, it proves to be useful to develop a linkage between the CFBM and the mode-matching technique. Since the CFBM is an extension of the boundary element method, the efficient boundary element method (BEM) solution is a sub-case of the CFBM. For continuity and briefness of the discussion, the general CFBM matrix equation[4,5] is rewritten here without detailed explanations:

$$\begin{bmatrix} [H_0 & H_1 & H_2 & H_{\Gamma_Q} & 0] & -[G_0 & G_1 & G_2 & G_{\Gamma_Q}] \\ 0 & 0 & 0 & A_{\Gamma_Q\Gamma_Q} & A_{\Gamma_Q Q} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{Q\Gamma_Q} & A_{QQ} & 0 & 0 & 0 & D \\ 0 & 1 & 0 & 0 & 0 & 0 & -Z_1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -Z_2 & 0 \end{bmatrix} \begin{Bmatrix} \{u\}_{\Gamma_0} \\ \{u\}_{\Gamma_1} \\ \{u\}_{\Gamma_2} \\ \{u\}_{\Gamma_Q} \\ \{u\}_Q \\ \{\frac{\partial u}{\partial n}\}_{\Gamma_0} \\ \{\frac{\partial u}{\partial n}\}_{\Gamma_1} \\ \{\frac{\partial u}{\partial n}\}_{\Gamma_2} \\ \{\frac{\partial u}{\partial n}\}_{\Gamma_Q} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 2\delta_{ij} f_{ik} \\ 0 \\ 0 \end{Bmatrix} \qquad (1)$$

$$i = 1, 2 \; ; \qquad k = (0), 1, 2, \ldots, N$$

where N is the number of mode that will be considered at port i, the indexes of the sub-matrixes 0 , 1, 2, and Γ_Q refers to the boundary consisting of conductor, ports and dielectric block, respectively. The details concerning the sub-matrixs H_i , G_j , A_{mn} and Z_l can be found in reference papers [4,5], and is therefore omitted here. In Eq.(1), the appropriate boundary conditions must be applied, that is, $\{u\}_{\Gamma_0} = \{0\}$ and $\{\partial u / \partial n\}_{\Gamma_0} = \{0\}$, for H-plane and E-plane circuits respectively.

Once the total fields across the waveguide ports are found by solving Eq.(1); the general scattering matrix is easily obtained as follows:

$$S_{jj}^{nn} = \pm\left[\int_{portj} u_n^j(x^{(j)}{=}0,\, y^{(j)}) f_{jn}\, dy^{(j)} - 1 \right] \qquad (2)$$

$$S_{ij}^{nm} = \pm\sqrt{\frac{k_{zn}^i}{k_{zm}^j}} \int_{porti} u_m^i(x^{(i)}{=}0,\, y^{(i)})\, f_{in}\, dy^{(i)} \qquad (3)$$

where u_m^i is the field solution across the i-th waveguide port by the excitation of the m-th mode at port j, f_{jn} is the mode functions, the ± sign depends on the direction of the contour for both the input and output ports used in the numerical analysis, as well as the index of the mode of interest.

It can be observed that, the matrix in Eq.(1) is independent of the mode function. Therefore, the matrix equation needs only to be solved once to obtain multi-mode information. Moreover, as long as the total fields at the ports are accurate, the higher order mode information can be extracted without any relative convergence problems. This feature is important when cascading numerically generated general scattering matrices with those obtained by the mode matching technique.

The mode matching solutions:

If a TE_{10} wave is incident at the initial input port of a waveguide circuit, and one considers the E-plane waveguide discontinuity case first, as shown in Fig.1, only the longitudinal section TE_{1m}^x mode will be excited. Using the convential mode matching procedure, one can have the matrix equation as

$$\underbrace{\begin{bmatrix} I & \sqrt{\dfrac{k_{zh\,1k}^I}{k_{zh\,1m}^{II}}}\, V_{km} \\ \sqrt{\dfrac{k_{zh\,1m}^I}{k_{zh\,1k}^{II}}}\, V_{mk} & -I \end{bmatrix}}_{\mathbf{P}} \begin{Bmatrix} a_{h\,1m}^I \\ a_{h\,1m}^{II} \end{Bmatrix} = \underbrace{\begin{bmatrix} I & \sqrt{\dfrac{k_{zh\,1k}^I}{k_{zh\,1m}^{II}}}\, V_{km} \\ -\sqrt{\dfrac{k_{zh\,1m}^I}{k_{zh\,1k}^{II}}}\, V_{mk} & I \end{bmatrix}}_{\mathbf{Q}} \begin{Bmatrix} b_{h\,1m}^I \\ b_{h\,1m}^{II} \end{Bmatrix} \qquad (4)$$

where

$$V_{km} = \int_{d_2}^{b_1+d_2} e_{h\,1k}^I\, e_{h\,1m}^{II}\, dy \qquad (5)$$

and $e_{h\,1m}^{\nu}$ is the mode function for TE_{1m}^x mode in sub-region v:

$$e_{h\,1m}^I = \sqrt{\frac{(2-\delta_{om})}{b_1}} \cos(m\pi(y-d_2)/b_1) \qquad (6)$$

$$e^{II}_{h1m} = \sqrt{\frac{(2-\delta_{om})}{b_2}}\cos(m\pi y/b_2) \qquad (7)$$

where δ_{0m} is the Kronecher delta.

Referring to the above, it follows that for the case of an H-plane waveguide step discontinuity, only the TE_{m0} mode needs to be considered. The matrix equation resulting from the mode matching procedure is

$$\underbrace{\begin{bmatrix} I & \sqrt{\dfrac{k^{II}_{zhk0}}{k^{I}_{zhn0}}}\,U_{kn} \\ \sqrt{\dfrac{k^{II}_{zhn0}}{k^{I}_{zhk0}}}\,U_{nk} & -I \end{bmatrix}}_{\mathbf{P}} \begin{Bmatrix} a^{I}_{hn0} \\ a^{II}_{hn0} \end{Bmatrix} = \underbrace{\begin{bmatrix} I & \sqrt{\dfrac{k^{II}_{zhk0}}{k^{I}_{zhn0}}}\,U_{kn} \\ -\sqrt{\dfrac{k^{II}_{zhn0}}{k^{I}_{zhk0}}}\,U_{nk} & I \end{bmatrix}}_{\mathbf{Q}} \begin{Bmatrix} b^{I}_{hn0} \\ b^{II}_{hn0} \end{Bmatrix} \qquad (8)$$

with

$$U_{kn} = \int_{c_2}^{c_2+a_1} e^{I}_{hk0}\, e^{II}_{hn0}\, dx \qquad (9)$$

and the mode function for the TE_{n0} mode in sub-region ν ($\nu = I$ or II) is

$$e^{I}_{hn0} = \sqrt{\frac{2}{a_1}}\sin(n\pi(x-c_2)/a_1) \qquad (10)$$

$$e^{II}_{hn0} = \sqrt{\frac{2}{a_2}}\sin(n\pi x/a_2) \qquad (11)$$

Then, it follows that the scattering matrix of a step discontinuity is given by

$$\mathbf{S} = \mathbf{Q}^{-1}\cdot\mathbf{P} \qquad (12)$$

3. Numerical Results and Discussions

In this section, a numerical example is presented to show the details for applying this technique in practice. The convergence of the solution is guaranteed by increasing the number of elements, especially the number of elements used at the waveguide ports, and the number of modes. In the numerical example, the second order element is used.

Fig.2 shows the transmission coefficient of a waveguide type dielectric filter. This example has been investigated by means of the FEM, as well as experimental measurements. As shown in Fig.3, the filter can be divided into three segments. D1 and D3 are regularly shaped step discontinuities; therefore they can be analyzed using the mode matching technique. D2 is composed of a dielectric block and is difficult to solve using analytical methods. Therefore, we resort to using the CFBM to generate the general scattering matrix for D2. The final result is obtained by cascading the general scattering matrices for D1, D2 and D3. The result obtained by full CFBM completely coincides with the result of the new schema, and thus is not shown in the figure. The experimental result given in [6] is superposed on the results obtained using the proposed schema. Good agreement is observed between these two sets of results.

4. Conclusions

An arbitrarily configured waveguide circuit can be segmented into; (1) regularly shaped discontinuities, (2) irregularly shaped discontinuities and (3) sections of homogeneous waveguide between discontinuities. The new schema proposed here is to analyze part (1) using an efficient analytical method, such as the mode matching technique, and part (2) using numerical tools. Part (3) is used to connect parts (1) and (2). It follows that since the coupling between higher order modes is taken into account through the general scattering matrix, the overall analysis remains rigorous.

In order to make use of the multi-mode general scattering matrix, the numerical method, i.e. coupled finite and boundary element method or boundary element method, has been extended to generate higher order mode information. This extension makes it possible to couple the analytical and numerical methods together.

References

[1] A. Wexler, "Solution of waveguide discontinuities by modal analysis," *IEEE Trans. Microwave Theory Tech.*, . **MTT-15**, pp.508-517, 1967.

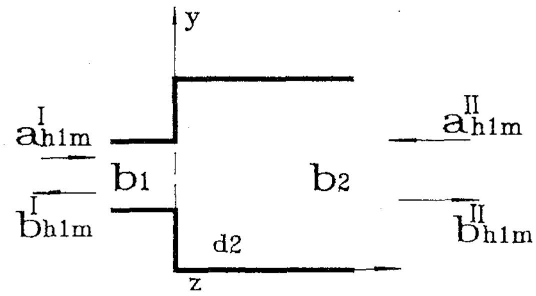
[2] M. Koshiba, M. Suzuki, "Finite-element analysis of H-plane waveguide junction with arbitrarily shaped ferrite post," *IEEE Trans. Microwave Theory Tech.*, . **MTT-34**, pp.103-109, 1986.

[3] M. Koshiba, M. Suzuki, "Application of the boundary-element method to waveguide discontinuities," *IEEE Trans. Microwave Theory Tech.*, . **MTT-34**, pp.301-307, 1986.

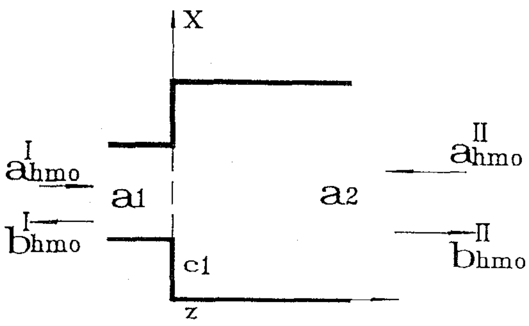
[4] K. L. Wu, G. Y. Delisle, D. G. Fang and M. Lecours, "Waveguide discontinuity analysis with a coupled finite-boundary element method," *IEEE Trans. Microwave Theory Tech.*, . **MTT-37**, pp.993-998, 1989.

[5] K. L. Wu, C. Wu and J. Litva, "Characterizing microwave planar circuits using the coupled finite-boundary element method," *IEEE Trans. Microwave Theory Tech.,* . **MTT-40**, pp.1963-1966, 1992.

[6] M. Koshiba, M. Sato, and M. Suzuki, "Application of finite-element method to H-plane waveguide discontinuities," *Electronics Letters,* vol. 18, No. 9, pp.364-365, 1982.



E–Plane waveguide discontinuity



H–Plane waveguide discontinuity

Fig.1 Two-dimensional waveguide discontinuities.

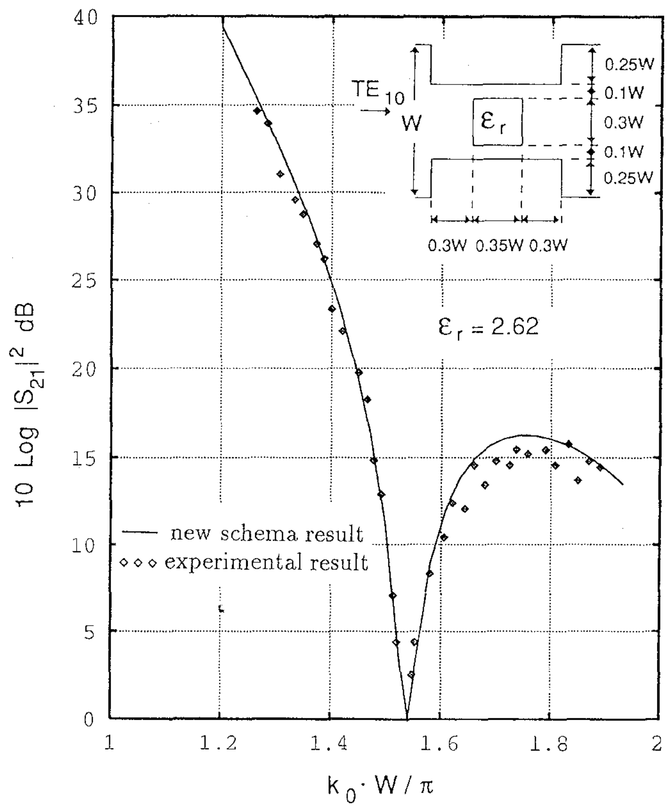


Fig.2 Power transmission coefficient of an H-Plane waveguide-type dielectric filter.

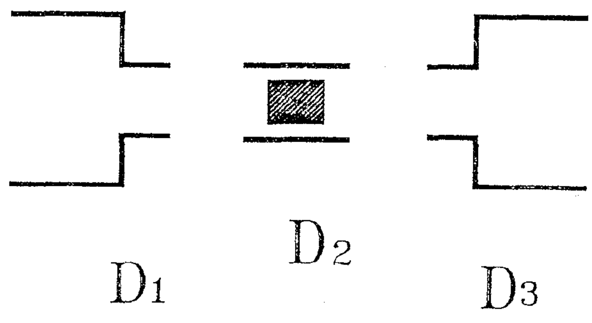


Fig.3 Segmented structure diagram for the waveguide-type dielectric filter.